

# Compressed Series for $e$

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(This notebook is referenced in the paper entitled "Improving the convergence of Newton's series approximation for  $e$ ," appearing in *The College Mathematics Journal*, Vol. 35, No. 1, 2004; pages 34 - 39.)

The following code takes an arbitrary number of terms and shows their compressed series along with the approximate numerical error relative to  $e$ . Here  $TC$  represents the number of terms combined (from Newton's series) into a single term of the new series, while  $t$  sets the number of these new terms to be evaluated.

Referring page: [www.brotherstechnology.com/math/cmj-sup.html](http://www.brotherstechnology.com/math/cmj-sup.html)  
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## Series for $e$

```
Clear[k]; TC = 2; t = 20;
numerat[1] = 1; numerat[TC_] := numerat[TC - 1] * (n + TC - 1) + 1;
denominat = (n + TC - 1)!;
summand = Together[Expand[numerat[TC] / denominat]];
expressn = summand /. n -> (TC * k); Print["e =  $\sum_{k=0}^{\infty}$ ", TraditionalForm@expressn]
k = t; acc = Floor[N[-Log[10, expressn]]] + 15;
Print["Error for ", t, " terms  $\approx$  ", E - N[Sum[expressn, {k, 0, t - 1}], acc]]
```

## Series for $e^x$

```
Clear[k, x]; TC = 2; t = 20;
numerat[1] = x^n; numerat[TC_] := numerat[TC - 1] * (n + TC - 1) + x^(n + TC - 1);
denominat = (n + TC - 1)!;
summand = Together[Expand[numerat[TC] / denominat]];
pwexpressn = summand /. n -> (TC * k); Print["e^x =  $\sum_{k=0}^{\infty}$ ", TraditionalForm@pwexpressn]
k = t; acc = Floor[N[-Log[10, pwexpressn]]] + 15; x = 1 / 2;
Print["Error for ", t, " terms  $\approx$  ", E - N[Sum[pwexpressn, {k, 0, t - 1}]^1/x, acc]]
```