To “e” or Not To “e”? That’s a Constant Question

By Daniel Willson
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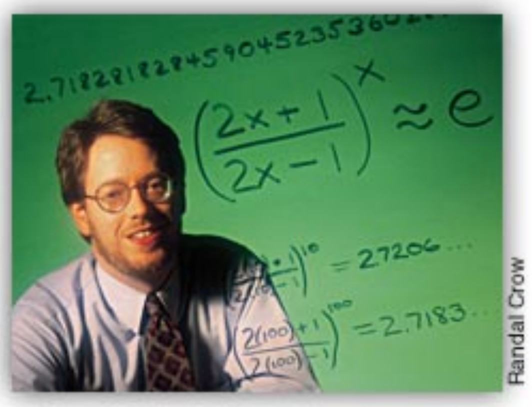
Trick question: What isn’t governed by or related to the universal mathematical constant *e*?



UAB alumnus John Knox (above), along with amateur mathematician Harlan Brothers, recently discovered a new formula to compute the mathematical constant *e*.

Answer: Probably nothing. If something grows or decays, chances are that *e*—an irrational constant number that is the base for natural logarithms—plays a vital role in the mathematical expressions that define those processes.

Now if the notion of an “irrational constant number” is starting to make you scratch your head, relax. Unless you’re a mathematician, all you need to know about *e* is that it works behind the scenes on a constant basis to quantify such phenomena as weather patterns, population growth, and radioactive decay. It even crops up in the financial world, defining the way that compound interest is calculated.

Another fact worth knowing about *e* is that a UAB alumnus and an amateur mathematician recently discovered a new and elegant formula to compute it. Thanks to their discovery, this little number could become an easily accessible concept for mathematics students and others who are intimidated by the complexities lurking in numerical expressions.

Musing on Multidimensional Space

This isn’t your typical story about research that culminated after years of toil in a laboratory or in front of a computer screen. Connecticut inventor Harlan Brothers was preparing for bed a few years ago, and he was doing what comes naturally when he needs to relax and clear his mind—thinking about prime numbers and multidimensional space. With pencil and paper in hand, Brothers began writing and evaluating several simple expressions. After immersing himself in this process for several minutes, he realized that he had apparently stumbled across an undiscovered method of approximating *e.* After further evaluation and an attempt to find the relationship between the classical method of calculating *e* and this new method, he discovered a second new equation for calculating the constant.

Brothers—who studied jazz guitar at the Berklee College of Music and currently holds five U.S. patents—decided to share his discoveries in early 1997 with Ian Stewart, Ph.D., of *Scientific American* magazine, and with Ira Flatow of National Public Radio’s (NPR) “Science Friday” program. While Brothers never received a response from *Scientific American,* his discoveries caught the attention of an NPR intern, who decided that Brothers’s letter was important enough to share with her husband.

A Serendipitous Scoop

Pam Naber Knox, a climatologist with a background in math and physics, was the intern with a keen eye for relevant science, and she decided that her husband, John, was the right person to examine Brothers’s findings.

At the time, John Knox was a postdoctoral fellow at Columbia University in New York City, working in association with the NASA/Goddard Institute for Space Studies. Using the knowledge he gained while an undergraduate mathematics student at UAB, Knox quickly found that Brothers’s findings were valid.

“It has to be one of the most unbelievable and circuitous stories you’ve ever heard about research,” says John Knox, Ph.D., who was the first student recruited for **UAB’s Honors Program** and the first UAB graduate awarded a National Science Foundation graduate fellowship. Currently an assistant professor of geography and meteorology at Valparaiso University in Indiana, Knox emphasizes that he’s “a meteorologist, not a mathematician. I’ve never had a number theory class in my life. My collaborator never studied in a college-level mathematics course, yet taught himself calculus. And my wife made the whole thing happen when she passed

Harlan's work along to me.

"Pam knew that I take pride in working with underdogs," Knox continues. "I fully expected to sit down, crunch a few numbers, find where Harlan's error was, and say, 'You're crazy for trying to scoop Isaac Newton and the other mathematical heavyweights.' But I couldn't find any errors. Still, I didn't think he had scooped anyone. So I wrote him, and we got together.

"Within two months, we had come up with several new formulas that I couldn't find in any calculus or number theory text. And by the fall of 1998, we published our findings in the second journal that we contacted."

Newer than Newton

The formulas of Knox and Brothers appeared in the October 1998 issue of *The Mathematical Intelligencer,* where the duo presented several elegant new algebraic expressions that had gone undiscovered by mathematicians for centuries.

The new theorems were devised using simple college-level calculus, says Knox, and they can be easily calculated on hand-held calculators. They are also much easier to grasp than traditional methods of teaching the constant, he says. "The best traditional method is a rather complicated computation known as the Taylor series," he explains. "It's based on work originally published in 1669 by Isaac Newton in the scholarly work, *De Analysi.*"

Like most new discoveries, the formulas of Knox and Brothers have drawn their fair share of critics—as well as supporters. University of Toronto mathematician Chandler Davis, Ph.D., editor of *The Mathematical Intelligencer,* says, "The experts 'knew' some old, familiar ways of computing *e,* and they knew that these converged in a very satisfactory way. That left it to two relative outsiders to search for devices to get qualitatively better convergence. Their search succeeded spectacularly."

On the other hand, numerical analyst Simon Plouffe of Hydro-Quebec in Montreal, who holds several computation records, states that other methods may be better for calculating *e;* he says the methods of Knox and Brothers would require too much computer memory to challenge the most accurate approximation of *e*—known to be accurate to 50 million decimal places.

Other inquiries about their work have come from as far away as Romania and Germany. German researcher Sebastian Wedeniwski now claims to hold the computational record for calculating *e* to more than 200 million places. He says that one of his three reasons for embarking on such a tedious task was to start testing Knox's and Brothers's theorems.

Doors to Discovery

"Our work is important because *e* is important," says Brothers. "We're not claiming that these theorems represent an advance in the computation of *e*—we've just come up with alternate formulas that may be easier to use in some circumstances. Regardless of whether our work has any actual practical applications, it is already having an impact by sparking the interest of teachers, students, and math buffs around the world. I find this very exciting."

Looking back on their discoveries, Knox says it is the sequence of events that still astonishes him. "'Serendipity' sums up how I, as a meteorologist with research interests in the stratosphere and in climate modeling, found myself collaborating with an inventor to discover new and elegant methods to calculate *e.*"

As for further collaborations, Knox and Brothers have co-authored another paper that appeared in the September 1999 issue of *College Mathematics Journal.* They are hopeful that the paper will encourage educators to use the new theorems in classrooms.

"Our simple but rigorously derived formulas should give high-school and college math teachers a golden opportunity to motivate their students," Knox says. "If two amateurs can scoop mathematical giants like Newton and Euler and come up with formulas that an eighth-grader can use, then imagine what else is out there to be discovered!"

More information about the new theorems is available here.